NARSwiki讲解系列（二）：Logic
——Higher-Order Inference (NALs-5)
Contents

1、Review
2、Higher-Order Inference (NALs-5)
3、Conclusion
1, Review

1. Entry points
   - user guides
   - example and demonstration (single step and multi-step)
   - Overview (Glossary)

First-Order Inference (NALs 1–4)

NAL-1
   - Inheritance copula
   - Revision, Choice, Deduction, Induction, Abduction, Conversion, Exemplification

NAL-2
   - Similarity, instance, property copula
   - Comparison, Analogy, Resemblance

NAL-3
   - Compound Terms

NAL-4
   - Ordinary relation
Contents

1、Review
2、Higher-Order Inference (NALs-5)
3、Conclusion
2. Higher-Order Inference (NALs-5)

NARS Wiki体系结构介绍

1. Entry points
   user guides
   example and demonstration (single step and multi-step)
   overview

2 Aspects of OpenNARS
Language and I/O
Logic+example and demonstration (single step and multi-step)
Data structure
Control
2、Higher-Order Inference (NALs-5)

NARS Wiki-logic

- Non-Axiomatic Logic
- Basic Inference in OpenNARS
- Sets and set operation in OpenNARS
  - Statements and Variables
  - Revision and choice rules
  - Variable, examples
  - Truth function
  - Basic syllogistic rules
  - Extended Boolean function
  - Compositional rules
  - Structural rules
  - Temporal inference
  - Procedural Inference
  - Introspective inference
  - Backward inference
2、Higher-Order Inference (NALs-5)

NARS Wiki—logic

- Statements and Variables
- Revision and choice rules
- Variable, examples
- Truth function
- Basic syllogistic rules
Concept
- Statements
- Term as Statements
  \(<(*, \{\text{Peter}\}, \langle \text{sky} \rightarrow \text{[blue]} \rangle) \Rightarrow \text{say}\). \%f, \%c

Copula
- implication \((\Rightarrow)\) if then
- equivalence \((\Leftrightarrow)\) if and only if
<table>
<thead>
<tr>
<th>Copula</th>
<th>NAL-1</th>
<th>NAL-2</th>
<th>NAL-3</th>
<th>NAL-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inheritance</td>
<td>Similarity</td>
<td>Intersection</td>
<td>Product</td>
</tr>
<tr>
<td></td>
<td>Instance</td>
<td>Difference</td>
<td>property</td>
<td></td>
</tr>
<tr>
<td>Rules</td>
<td>Revision</td>
<td>Comparison</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Choice</td>
<td>Analogy</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Deduction</td>
<td>Resemblance</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Induction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Abduction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conversion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exemplificatio</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Higher-Order Inference (NALs–5)

<table>
<thead>
<tr>
<th>First-Order</th>
<th>Higher-Order</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>term</em></td>
<td>statement</td>
</tr>
<tr>
<td><em>inheritance</em></td>
<td><em>implication</em></td>
</tr>
<tr>
<td><em>similarity</em></td>
<td><em>equivalence</em></td>
</tr>
<tr>
<td><em>subject</em></td>
<td><em>antecedent</em></td>
</tr>
<tr>
<td><em>predicate</em></td>
<td><em>consequent</em></td>
</tr>
<tr>
<td><em>extension</em></td>
<td><em>sufficient condition</em></td>
</tr>
<tr>
<td><em>intension</em></td>
<td><em>necessary condition</em></td>
</tr>
<tr>
<td><em>extensional intersection</em></td>
<td><em>conjunction</em></td>
</tr>
<tr>
<td><em>intentional intersection</em></td>
<td><em>disjunction</em></td>
</tr>
</tbody>
</table>
Higher-Order Inference (NALs-5)

Deduction
// If robin is a type of bird then robin is a type of animal.
\langle\text{robin }\rightarrow \text{ bird} \rangle \Rightarrow \langle\text{robin }\rightarrow \text{ animal}\rangle

// If robin can fly then robin is a type of bird.
\langle\text{robin }\rightarrow \text{ [flying]} \rangle \Rightarrow \langle\text{robin }\rightarrow \text{ bird}\rangle

// If robin can fly then robin is a type of animal.
//outputMustContain("\langle\text{robin }\rightarrow \text{ [flying]} \rangle \Rightarrow \langle\text{robin }\rightarrow \text{ animal}\rangle. \%1.00;0.81\%")
Higher-Order Inference (NALs-5)

**Induction**

// If robin is a type of bird then robin is a type of animal.

\[
\langle \langle \text{robin} \rightarrow \text{bird} \rangle \Rightarrow \langle \text{robin} \rightarrow \text{animal} \rangle \rangle.
\]

// If robin is a type of bird then robin can fly.

\[
\langle \langle \text{robin} \rightarrow \text{bird} \rangle \Rightarrow \langle \text{robin} \rightarrow \text{[flying]} \rangle \rangle. \%0.80\
\]

// I guess if robin can fly then robin is a type of animal.

// outputMustContain

\[
\'\langle \langle \text{robin} \rightarrow \text{[flying]} \rangle \Rightarrow \langle \text{robin} \rightarrow \text{animal} \rangle \rangle. \%1.00;0.39\'\]

// I guess if robin is a type of animal then robin can fly.

// outputMustContain

\[
\'\langle \langle \text{robin} \rightarrow \text{animal} \rangle \Rightarrow \langle \text{robin} \rightarrow \text{[flying]} \rangle \rangle. \%0.80;0.45\'\]
2. Higher-Order Inference (NALs-5)

**Abduction**

// If robin is a type of bird then robin is a type of animal.

\(<\langle\text{robin} \rightarrow \text{bird}\rangle \implies \langle\text{robin} \rightarrow \text{animal}\rangle\>\).

// If robin can fly then robin is probably a type of animal.

\(<\langle\text{robin} \rightarrow \text{[flying]}\rangle \implies \langle\text{robin} \rightarrow \text{animal}\rangle\>\). %0.80

// I guess if robin is a type of bird then robin can fly.

// outputMustContain

(’\(<\langle\text{robin} \rightarrow \text{bird}\rangle \implies \langle\text{robin} \rightarrow \text{[flying]}\rangle\>\). %1.00;0.39’)

// I guess if robin can fly then robin is a type of bird.

// outputMustContain

(’\(<\langle\text{robin} \rightarrow \text{[flying]}\rangle \implies \langle\text{robin} \rightarrow \text{bird}\rangle\>\). %0.80;0.45’)

\[\begin{array}{c}
P \rightarrow M(f_1, c_1) \\
S \rightarrow M(f_2, c_2) \\
S \rightarrow P(f, c) \end{array}\]
Exemplification (Inverse deduction)
//If robin can fly then robin is a type of bird.
\langle\langle\text{robin} \rightarrow [\text{flying}]\rangle \Rightarrow \langle\text{robin} \rightarrow \text{bird}\rangle\rangle.

//If robin is a type of bird then robin is a type of animal.
\langle\langle\text{robin} \rightarrow \text{bird}\rangle \Rightarrow \langle\text{robin} \rightarrow \text{animal}\rangle\rangle.

I guess if robin is a type of animal then robin can fly.

//outputMustContain
(’\langle\langle\text{robin} \rightarrow \text{animal}\rangle \Rightarrow \langle\text{robin} \rightarrow [\text{flying}]\rangle\rangle.
\%1.00;0.45’)

\[P \rightarrow M(f_1, c_1)\]
\[M \rightarrow S(f_2, c_2)\]
\[S \rightarrow P(f, c)\]
Comparison
//If robin is a type of bird then robin is a type of animal.
\langle\langle\text{robin} \rightarrow \text{bird}\rangle \implies \langle\text{robin} \rightarrow \text{animal}\rangle\rangle.

//If robin is a type of bird then robin can fly.
\langle\langle\text{robin} \rightarrow \text{bird}\rangle \implies \langle\text{robin} \rightarrow \text{[flying]}\rangle\rangle. \%0.80\%

//I guess robin is a type of animal if and only if robin can fly.
//outputMustContain
\langle\langle\text{robin} \rightarrow \text{[flying]}\rangle \iff \langle\text{robin} \rightarrow \text{animal}\rangle\rangle. \%0.80;0.45\%
Higher-Order Inference (NALs-5)

Analogy
// If robin is a type of bird then robin is a type of animal.
\langle \text{robin} \rightarrow \text{bird} \rangle \implies \langle \text{robin} \rightarrow \text{animal} \rangle.

// Usually, robin is a type of bird if and only if robin can fly.
\langle \text{robin} \rightarrow \text{bird} \rangle \iff \langle \text{robin} \rightarrow [\text{flying}] \rangle. \%0.80%

// If robin can fly then probably robin is a type of animal.

// outputMustContain
('\langle \text{robin} \rightarrow [\text{flying}] \rangle \implies \langle \text{robin} \rightarrow \text{animal} \rangle. \%0.80; 0.65\%')
Resemblance

//Robin is a type of animal if and only if robin is a type
\langle\langle\text{robin} \rightarrow \text{animal}\rangle \iff \langle\langle\text{robin} \rightarrow \text{bird}\rangle\rangle\rangle.

//Robin is a type of bird if and only if robin can fly.
\langle\langle\text{robin} \rightarrow \text{bird}\rangle \iff \langle\langle\text{robin} \rightarrow \text{[flying]}\rangle\rangle\rangle. \%0.9\%

//Robin is a type of animal if and only if robin can fly.

//outputMustContain(
'\langle\langle\text{robin} \rightarrow \text{[flying]}\rangle \iff \langle\langle\text{robin} \rightarrow \text{animal}\rangle\rangle\rangle. \%0.90;0.81\')
Negation
	negation of a statement is a compound term with positive and negative evidence switched.

For the statement \( <S> \%f_0; c_0\% \), its negation is \( <(--, S)> \%f_1; c_1\% \)

where negation truth function is then defined as:

\[
f = 1 - f_0, \quad c = c_0
\]

Contrapositive: Important to note that Law of **Contrapositive** \((S \Rightarrow T \equiv \neg T \Rightarrow \neg S)\) is no longer true, therefore NAL-5 introduces another variant of conversion rule from NAL-1 that is from

\[<S_1 \Rightarrow S_2> \%f_0; c_0\%
\]

NAL derives

\[<(--, S_2) \Rightarrow (--, S_1)> \%f_1; c_1\%
\]

where truth value is computed using

\[
f = 0, \quad c = (1-f_0)c_0/(f_0c_0 + 1)
\]
2、Higher-Order Inference (NALs-5)

NARS Wiki-logic

- Statements and Variables
- Revision and choice rules
- Variable, examples
- Truth function
- Basic syllogistic rules
Revision and choice rules

Revision is an inference process in which evidence from different sources is being combined and its truth value revised. Suppose there are two tasks with different truth values, for example:

\(<\text{snow} \rightarrow \{\text{white}\}>. \ 9/10, 10/11\%
\(<\text{snow} \rightarrow \{\text{white}\}>. \ 3/4, 4/5\%

\[
f = \frac{f_1 c_1 (1 - c_2) + f_2 c_2 (1 - c_1)}{c_1 (1 - c_2) + c_2 (1 - c_1)}
\]

\[
c = \frac{c_1 (1 - c_2) + c_2 (1 - c_1)}{c_1 (1 - c_2) + c_2 (1 - c_1) + (1 - c_1)(1 - c_2)}
\]

Definition 3.7. A belief in the system is a judgment in its memory that is either an element of experience \(K\), or derived from some elements of \(K\). At a given moment, the collection of all beliefs is called the system’s knowledge \(K^*\). The evidential base of a belief is the set of beliefs in \(K\) from which the belief is derived.

- The evidential base of an input judgment is a set containing itself,
- the evidential base of a derived conclusion is the union of the evidential bases of the premises deriving the conclusion.
- Thus it is possible that after some derivation steps, early task’ serial numbers have been lost and it is exactly what the system tries to simulate, the concept of biological memory.
Revision and choice rules

Overlapping Evidence

For two tasks if they share one or more same elements within their evidential bases, their evidence is overlapped i.e. they do not have disjoint evidential bases.

Ideally when combining evidence, and if some portion of evidence is present more than once, it should be subtracted from the final result, however since it is impossible to determine the exact amount of some portion of evidence influencing certain task this can’t be done. Therefore revision rule applies only if tasks have disjoint evidential bases that are they do not share common elements.
Revision and choice rules
The choice rule is applicable in multiple scenarios.
- Two tasks are based on overlapping evidence and revision rule cannot be applied, then a choice must be made.
- Suppose there are two competing candidate answers \(<S \rightarrow A>\). \%f1, c2\% and \(<S \rightarrow B>\) \%f2, c2\% for the question ?1>? . These candidates may be of the same or different natures and will have different truth values. Thus it is no longer appropriate to choose the answer with higher confidence.
To handle such situations truth expectation metrics are introduced.
\[ e = c \times (f - 0.5) + 0.5 \]
The candidate with the higher truth expectation is selected by choice rule.
2、Higher-Order Inference (NALs-5)

NARS Wiki—logic

- Statements and Variables
- Revision and choice rules
- Variable, examples
- Truth function
- Basic syllogistic rules
Truth function

A typical simple inference cycle in OpenNARS takes two judgments as premise and derives a judgment as a conclusion. An example looks like:

\{\langle\text{premise1}\rangle. \%f1; c1\%, \langle\text{premise2}\rangle. \%f2; c2\%\} \quad |\quad -- \quad \langle\text{conclusion}\rangle. \%f; c\%
<table>
<thead>
<tr>
<th>type</th>
<th>inference</th>
<th>name</th>
<th>function</th>
</tr>
</thead>
</table>
| **local inference** | revision  | $F_{rev}$ | $w^+ = w_1^+ + w_2^+$  
|                   |           |      | $w^- = w_1^- + w_2^-$ |
|                   | expectation| $F_{exp}$ | $e = c(f - 0.5) + 0.5$ |
|                   | decision  | $F_{dec}$ | $g = p(d - 0.5)$       |
| **immediate inference** | negation  | $F_{neg}$ | $w^+ = w_1^-$  
|                   |           |      | $w^- = w_1^+$ |
|                   | conversion | $F_{cnv}$ | $w^+ = \text{and}(f_1, c_1)$  
|                   |           |      | $w^- = 0$ |
|                   | contraposition | $F_{cnt}$ | $w^+ = 0$  
|                   |           |      | $w^- = \text{and}((\text{not}(f_1), c_1)$ |
| **strong syllogism** | deduction | $F_{ded}$ | $f = \text{and}(f_1, f_2)$  
|                   |           |      | $c = \text{and}(f_1, f_2, c_1, c_2)$ |
|                   | analogy   | $F_{ana}$ | $f = \text{and}(f_1, f_2)$  
|                   |           |      | $c = \text{and}(f_2, c_1, c_2)$ |
|                   | resemblance | $F_{res}$ | $f = \text{and}(f_1, f_2)$  
|                   |           |      | $c = \text{and}(\text{or}(f_1, f_2), c_1, c_2)$ |
| **weak syllogism** | abduction | $F_{abd}$ | $w^+ = \text{and}(f_1, f_2, c_1, c_2)$  
|                   |           |      | $w = \text{and}(f_1, c_1, c_2)$ |
|                   | induction | $F_{ind}$ | $w^+ = \text{and}(f_1, f_2, c_1, c_2)$  
|                   |           |      | $w = \text{and}(f_2, c_1, c_2)$ |
|                   | exemplification | $F_{exe}$ | $w^+ = \text{and}(f_1, f_2, c_1, c_2)$  
|                   |           |      | $w = \text{and}(f_1, f_2, c_1, c_2)$ |
|                   | comparison | $F_{com}$ | $w^+ = \text{and}(f_1, f_2, c_1, c_2)$  
|                   |           |      | $w = \text{and}(\text{or}(f_1, f_2), c_1, c_2)$ |
| **term composition** | intersection | $F_{int}$ | $f = \text{and}(f_1, f_2)$  
|                   |           |      | $c = \text{and}(c_1, c_2)$ |
|                   | union     | $F_{uni}$ | $f = \text{or}(f_1, f_2)$  
|                   |           |      | $c = \text{and}(c_1, c_2)$ |
|                   | difference | $F_{dif}$ | $f = \text{and}(f_1, \text{not}(f_2))$  
|                   |           |      | $c = \text{and}(c_1, c_2)$ |
2. Higher-Order Inference (NALs-5)

NARS Wiki-logic

- Statements and Variables
- Revision and choice rules
- Variable, examples
- Truth function
- Basic syllogistic rules
2. Higher-Order Inference (NALs–5)

- Basic syllogistic rules

<table>
<thead>
<tr>
<th>$J_2 \setminus J_1$</th>
<th>$M \rightarrow P \langle f_1, c_1 \rangle$</th>
<th>$P \rightarrow M \langle f_1, c_1 \rangle$</th>
<th>$M \leftrightarrow P \langle f_1, c_1 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S \rightarrow M \langle f_2, c_2 \rangle$</td>
<td>$S \rightarrow P \langle F_{ded} \rangle$</td>
<td>$S \rightarrow P \langle F_{abd} \rangle$</td>
<td>$S \rightarrow P \langle F'_{ana} \rangle$</td>
</tr>
<tr>
<td>$P \rightarrow S \langle F_{exe} \rangle$</td>
<td>$P \rightarrow S \langle F'_{ded} \rangle$</td>
<td>$S \leftrightarrow P \langle F'_{com} \rangle$</td>
<td>$P \rightarrow S \langle F'_{ana} \rangle$</td>
</tr>
<tr>
<td>$M \rightarrow S \langle f_2, c_2 \rangle$</td>
<td>$S \rightarrow P \langle F_{ind} \rangle$</td>
<td>$S \rightarrow P \langle F_{exe} \rangle$</td>
<td>$P \rightarrow S \langle F'_{ana} \rangle$</td>
</tr>
<tr>
<td>$P \rightarrow S \langle F'_{ind} \rangle$</td>
<td>$P \rightarrow S \langle F'_{ded} \rangle$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S \leftrightarrow P \langle F_{com} \rangle$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S \leftrightarrow M \langle f_2, c_2 \rangle$</td>
<td>$S \rightarrow P \langle F_{ana} \rangle$</td>
<td>$P \rightarrow S \langle F_{ana} \rangle$</td>
<td>$S \leftrightarrow P \langle F_{res} \rangle$</td>
</tr>
</tbody>
</table>
Contents

1. Review
2. Higher-Order Inference (NALs-5)
3. Conclusion
3. Conclusion

- NAL–5
- Statements
- Negation
- Revision and choice
- Basic syllogistic rules and Truth function